Ends of nonpositively curved manifolds

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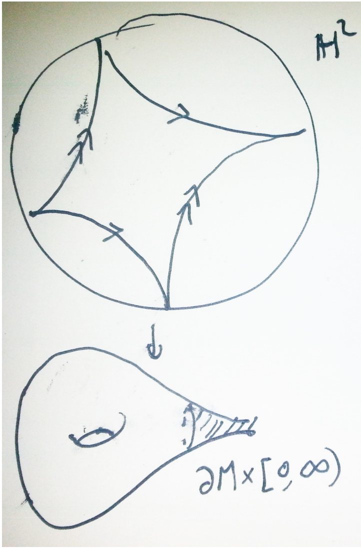
Non-compact hyperbolic surfaces



Hyperbolic manifolds

M complete, Riemannian, $vol < \infty$, curvature $K = -1$.

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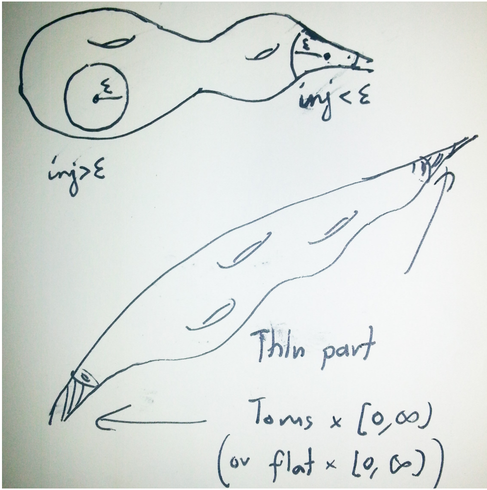
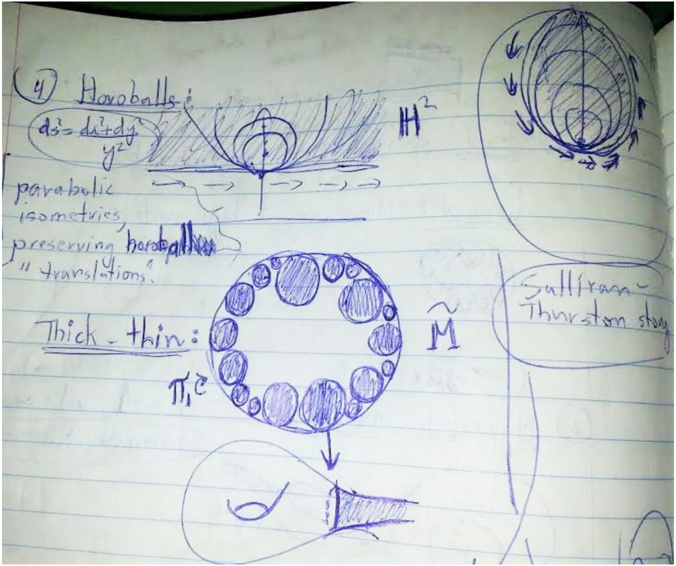
$inj(x) =$ max radius of embedded ball centered at x.



Pinched negative curvature $-1 < K < -\delta < 0$.

Same as hyperbolic, except ∂M is finitely covered by Nil manifold.

Horoballs



(4) Horoballs:
$ds^2 = \frac{dx^2+dy^2}{y^2}$
$\mathbb{H}^2$
parabolic isometries, preserving horoballs
" translations"
Thick - thin:
$\widetilde{M}$
$\pi_1 \curvearrowright$
Sullivan - Thurston story

Tameness

Theorem (Gromov)

If $-1 < K < 0$ and vol$(M) < \infty$ then M is the interior of a compact manifold with boundary ∂M.

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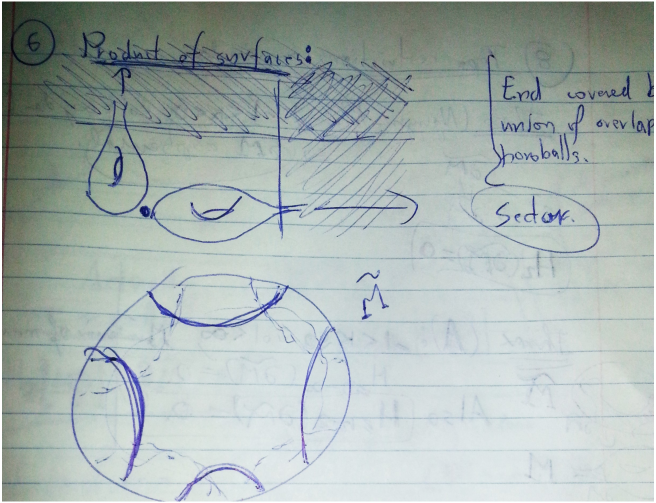
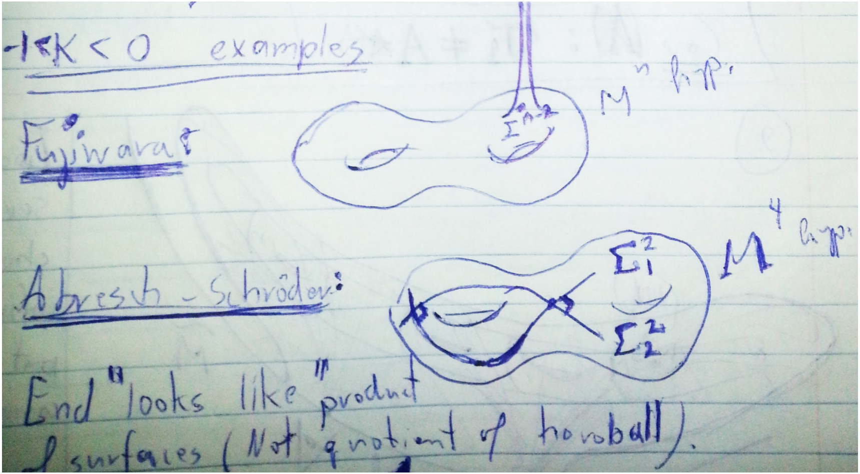
Question:

What is topology of ∂M? Is it aspherical? Is it a quotient of a horoball?

Products of surfaces



Negatively curved examples



Topological restrictions on ends

M complete, Riemannian manifold.

Theorem (Nguyen-Phan)

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Theorem (A.)

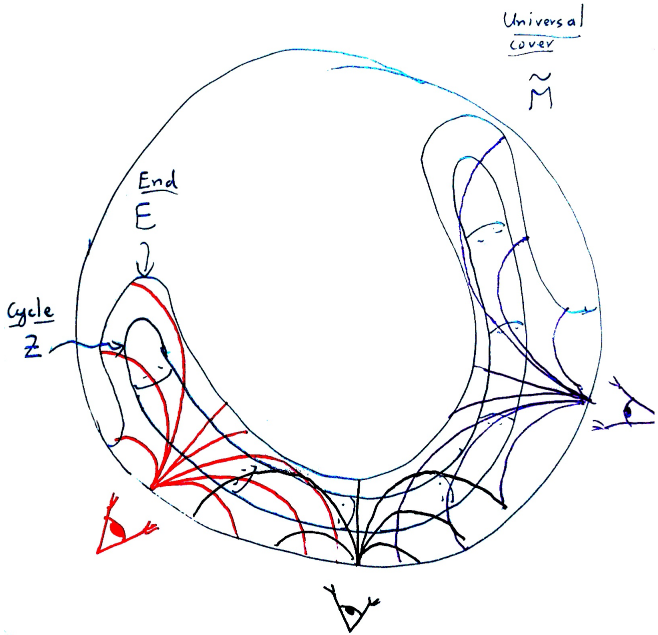
If $-1 < K < 0$ and $vol(M) < \infty$ then

$$H_{\geq n-2}(\widetilde{\partial M}) = 0, \tag{1}$$
$$H_{\geq n-2}(\partial \widetilde{M}) = 0. \tag{2}$$

The same is true if $-1 < K \leq 0$ and M is tame.

Corollary (A.)

The fundamental group is freely indecomposable: $\pi_1 M \neq A * B$.



Universal cover
$\tilde{M}$
End
E
Cycle
Z