Analytic torsion and dynamical zeta functions of locally symmetric spaces

Abstract

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A variety of topics in the field of spectral geometry are concerned with the study of the dynamical zeta functions of Ruelle and Selberg and their relation to spectral invariants such as the eta invariant associated with Dirac-type operators and the analytic torsion. The dynamical zeta functions are certain functions of a complex variable. They can be used as a tool to count the periodic orbits of a dynamical system. We consider the dynamical zeta functions, which are attached to the geodesic flow on the unit tangent bundle over a hyperbolic manifold.

In this talk, we present some recent results concerning the Selberg and Ruelle zeta functions on compact oriented hyperbolic manifolds X of odd dimension d. We identify X with $\Gamma\backslash G/K$, where $G = \mathrm{SO}^0(d,1)$, $K = \mathrm{SO}(d)$ and Γ is a discrete torsion-free cocompact subgroup of G. Let $G = KAN$ be the Iwasawa decomposition with respect to K. Let M be the centralizer of A in K.

For an irreducible representation σ of M and a finite dimensional representation χ of Γ, we define the Selberg zeta function $Z(s;\sigma,\chi)$ and the Ruelle zeta function $R(s;\sigma,\chi)$. We prove that they converge in some half-plane $\mathrm{Re}(s) > c$ and admit a meromorphic continuation to the whole complex plane. We also describe the singularities of the Selberg zeta function in terms of the discrete spectrum of certain differential operators on X. Furthermore, we provide functional equations relating their values at s with those at $-s$. The main tool that we use is the Selberg trace formula for non-unitary twists ([Mül11]). We generalize results of Bunke and Olbrich ([BO95]) to the case of non-unitary representations χ of Γ. Finally, we prove a determinant formula for the Ruelle zeta function, using the regularized determinant of the twisted Laplace-type operator $A^{\sharp}_{\tau,\chi}(\sigma)$, which is the key point to approach the Fried's conjecture, i.e., the relation of the Ruelle zeta funcion to the analytic torsion. The even dimensional case of hyperbolic manifolds will be also discussed.

Bibliography

[BO95] Ulrich Bunke and Martin Olbrich, *Selberg zeta and theta functions*, Mathematical Research, vol. 83, Akademie-Verlag, Berlin, 1995, A differential operator approach.

[Mül11] Werner Müller, *A Selberg trace formula for non-unitary twists*, Int. Math. Res. Not. IMRN (2011), no. 9, 2068–2109.